

CALCULATION OF A JET AXIS IN A DRIFTING FLOW

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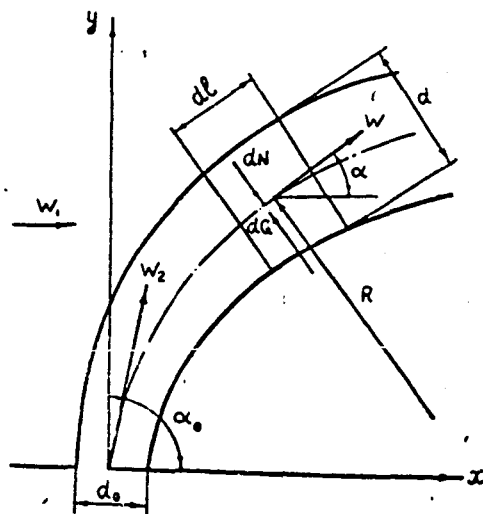
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РАСЧЕТ ОСИ СТРУИ В СНОСЯЩЕМ ПОТОКЕ

Г. С. ШАНДОРОВ

В литературе известен ряд работ по экспериментальному исследованию формы оси одиночной газовой струи в дозвуковом сносящем потоке. Результаты одного из таких исследований, выполненного в диапазоне определяющих параметров, характерных для камер сгорания авиационных газотурбинных двигателей, были опубликованы автором данной статьи в 1955 году [1].

В последнее время предприняты попытки теоретического расчета формы оси веерных и парных плоских струй в свободном и ограниченном сносящих потоках, основанные на рассмотрении сил, воздействующих на элементарный участок струи [2], [3].



Фиг. 1. Расчетная схема струи в сносящем потоке.

Покажем, что такой подход к решению задачи об изгибе струи приводит к удовлетворительным результатам и в случае одиночной, круглой в начальном сечении, сносимой струи, распространяющейся в однородном неограниченном дозвуковом потоке.

Для этого на некотором расстоянии от устья струи, расположенного в плоскости xOz , выделим ее элемент, протяженностью dl (фиг. 1). Будем считать, что изгибающая струю аэродинамическая сила пропорциональна скоростному напору нормальной составляющей скорости сносящего потока и уравнивается центробежной силой. Тогда условие радиального равновесия элемента струи dl запишется

$$dN = dQ, \quad (1)$$

где $dN = C_x \frac{\rho_1 w_1^2}{2} \sin^2 \alpha dl$, $dQ = \frac{\rho w^2}{R} \frac{\pi d^2}{4} dl$ — аэродинамическая и центробежная силы. Подставляя выражения для сил, действующих на элемент dl , в равенство (1), получим

$$C_x \frac{q_1}{q_2} \sin^2 \alpha = \frac{d_0}{R}, \quad (2)$$

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CALCULATION OF A JET AXIS IN A DRIFTING FLOW

G. S. Shandorov

The article discusses derivation of a solution for the problem of deflection of a jet which is subject to drift in a uniform unbounded subsonic gas flow. The solution is based on an analysis of forces acting on a jet element and is applicable to a single jet with an originally circular cross section.

A series of works is known in the literature on the experimental investigation of the form of the axis corresponding to a single gas jet in a subsonic drifting flow. The results of one such investigation performed in the region of characteristic parameters which are common for the combustion chambers of aviation gas turbines were published by the author in 1955 (ref. 1). /100*

Recently, efforts have been made to compute theoretically the form of the axis corresponding to plane fan and twin jets in free, drifting flows based on the consideration of forces which act on the elementary region of the jet (refs. 2 and 3).

We shall show that this approach to the solution of the problem on the inflection of the jet leads to satisfactory results in the case when we have a single drifting jet whose initial cross section is round and which propagates in a homogeneous infinite subsonic flow.

For this purpose we isolate a jet element having a length dl at some distance from the mouth of the jet, which is situated in the xoz plane (fig. 1). We shall assume that the aerodynamic force which bends the jet is proportional to the velocity head of the normal velocity component of the drift-producing flow and is counterbalanced by the centrifugal force. Then the condition for the radial equilibrium of the jet element dl may be written in the form

$$\text{wh } dN = C_x \frac{\rho_1 w_1^2}{2} \sin^2 \alpha dl, \quad dQ = \frac{v w^2}{R} \frac{\pi d^2}{4} dl \quad (1)$$

are the aerodynamic and centri-

fugal forces. Substituting the expressions for the forces acting on the element dl into equality (1) we obtain

$$C_x \frac{q_1}{q_2} \sin^2 \alpha = \frac{d_0}{R}, \quad (2)$$

where

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*Numbers given in margin indicate pagination in original foreign text.

(3)

q_1, q_2, q are the velocity heads of the flow producing the drift of the jet at the initial and running cross sections respectively. We carry out the substitutions

$$\sin \alpha = \frac{y'}{(1+y'^2)^{0.5}}, \quad R = \frac{(1+y'^2)^{1.5}}{y'}$$

Then equation (2) will have the form:

$$\frac{C_R}{d_0} \frac{q_1}{q_2} y'^2 = \frac{y''}{(1+y'^2)^{0.5}} \quad (4)$$

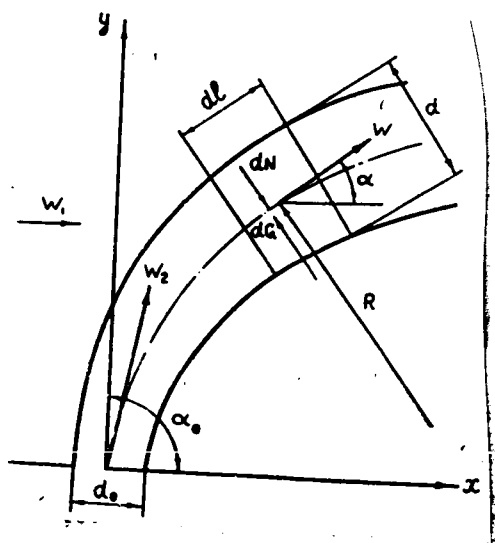


Figure 1. Schematic of the jet in the drifting flow.

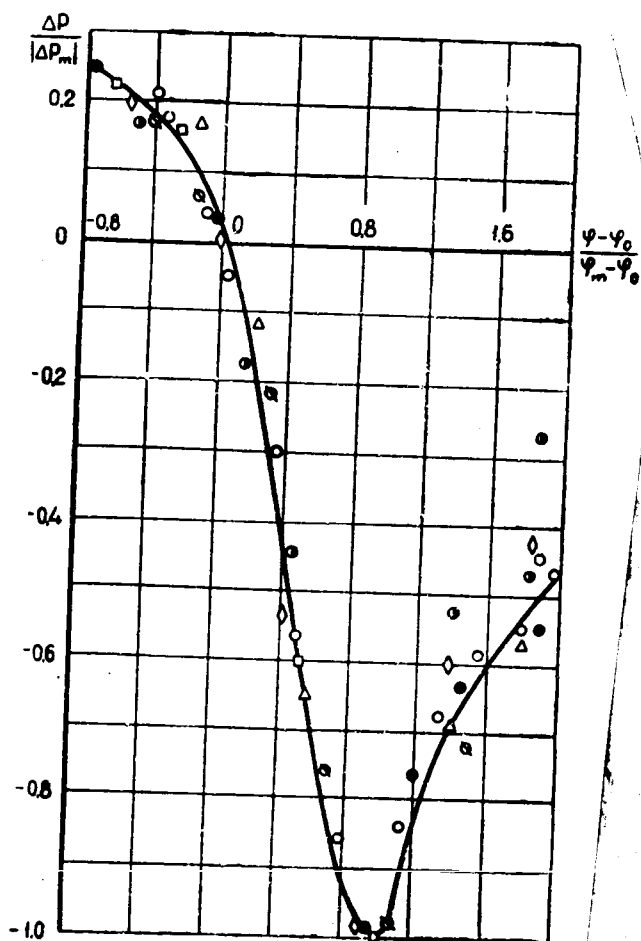


Figure 2. Pressure distribution around the initial cross section of the jet in terms of dimensionless coordinates:

Before integrating (4) we consider carefully the quantity C_n contained in this equation. In the expression for C_n (3), three generally speaking, three variable quantities C_x , q and d are contained in addition to parameters q_2 , d_0 .

It is known that as we move away from the mouth the decrease in the velocity head of the jet is accompanied by an increase in its cross section so that the effect of parameters q and d on the quantity C_n is compensated for to some extent. In other words, we may assume that: /102

(5)

As shown in reference 1, the pressure distribution around the jet in the drifting flow, which determines the quantity C_x when the mouth of the jet enters the flow at an angle $\alpha_0 = 90^\circ$, is a function of the ratio of jet and flow impact pressures. Figure 2 shows the pressure distribution profile around the initial section of the jet in the dimensionless coordinates:

$$\frac{\Delta p}{|\Delta p_m|}, \frac{\varphi - \varphi_0}{\varphi_m - \varphi_0}$$

Here Δp_m is the maximum rarefaction around the jet,

- Δp is the static pressure in excess of the pressure in the unperturbed drifting flow,
- φ is the angular coordinate of the considered point on the surface around the jet,
- φ_0 is the angular coordinate of the point at which the pressure is equal to the pressure of the drifting flow,
- φ_m is the angular coordinate of the point where the pressure is equal to Δp_m .

For the range of ratios $1.5 \leq q_2/q_1 \leq 15$ investigated in reference 1, the experimental points lie practically on one curve. In the rear part of the jet when $\varphi > \varphi_m$ the spread of the points is larger than when $\varphi < \varphi_m$, and in case $q_2/q_1 \leq 2$ there is a systematic departure from the curve due to the increased influence of the wall. For large velocity in the drift flow, the jet which exists from the nozzle "adheres" to the wall. The universal nature of pressure distribution must obviously be violated also when the velocities of the drift flow are very small, because as in the limiting case corresponding to the free jet, the excess pressures at all points of the cross section are the same and are equal to zero.

When using the universal profile for the pressure distribution around the jet, we must know the relationships $|\Delta p_m|/q_2$, φ_0 and φ_m as a function of the ratio q_2/q_1 . (These relationships, which have been determined experimentally, are shown in the graphs of figure 3.)

As we can see from the graphs, when the ratio q_2/q_1 changes, there is a /103 substantial deformation of the pressure diagram around the jet. The processing of presented experimental data shows, however, that when q_2/q_1 varies from 1.5 to 15, the value of C_x changes relatively little:¹ from 3.5 to 4.5. It should also be pointed out that in reference 1 the pressure distribution around the mouth of the jet was determined by draining the nozzle cutoff plane at a distance of 1.5 mm from the circumference of the output hole. Therefore we can expect a somewhat higher actual value for C_x , specifically a value of the order of 4.2 - 5.2. In this case the probable values of C_n according to (5) are equal to 2.7 - 3.3.

Assuming, in the first approximation, that the magnitude of C_n is constant along the length of the jet and is the same for different ratios q_2/q_1 , we can easily integrate the differential equation (4). As a result we obtain:

$$\frac{y}{d_0} = \frac{1}{C_n} \cdot \frac{q_2}{q_1} \cdot \ln 2 \left[\sqrt{\left(C_1 + C_n \frac{q_1}{q_2} \frac{x}{d_0}\right)^2 - 1} + C_1 + C_n \frac{q_1}{q_2} \frac{x}{d_0} \right] + C_2 \quad (6)$$

The constants of integration C_1 , C_2 are obtained from the boundary conditions:

$$\text{for } x = 0, y = 0, y' = \operatorname{tg} \alpha_0.$$

It follows that

$$C_1 = \frac{1}{\sin \alpha_0}, \quad C_2 = -\frac{1}{C_n} \cdot \frac{q_2}{q_1} \ln 2 \operatorname{ctg} \frac{\alpha_0}{2}.$$

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Substituting the values of C_1 , C_2 into equation (6), we finally obtain:

$$\frac{y}{d_0} = \frac{1}{C_n} \frac{q_2}{q_1} \ln \operatorname{tg} \frac{\alpha_0}{2} \left[\sqrt{\left(\frac{1}{\sin \alpha_0} + C_n \frac{q_1}{q_2} \frac{x}{d_0}\right)^2 - 1} + \frac{1}{\sin \alpha_0} + C_n \frac{q_1}{q_2} \frac{x}{d_0} \right]. \quad (7)$$

¹The large value of C_x is explained by the presence of rarefaction behind the drift jet which substantially exceeds the one produced on the surface of a solid cylinder when there is flow around it.

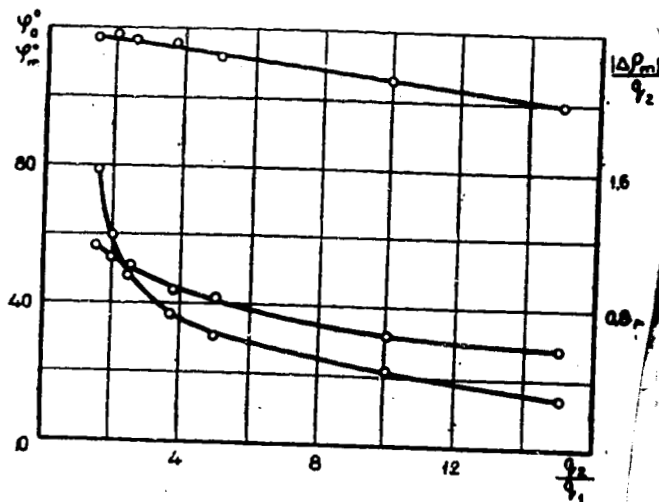


Figure 3. Variation in the maximum dimensionless rarefaction at the initial cross section of the jet $|\Delta p_m|/q_2$ and in the coordinates φ_0 and φ_m as a function of the ratio of impact heads in the jet and in the stream.

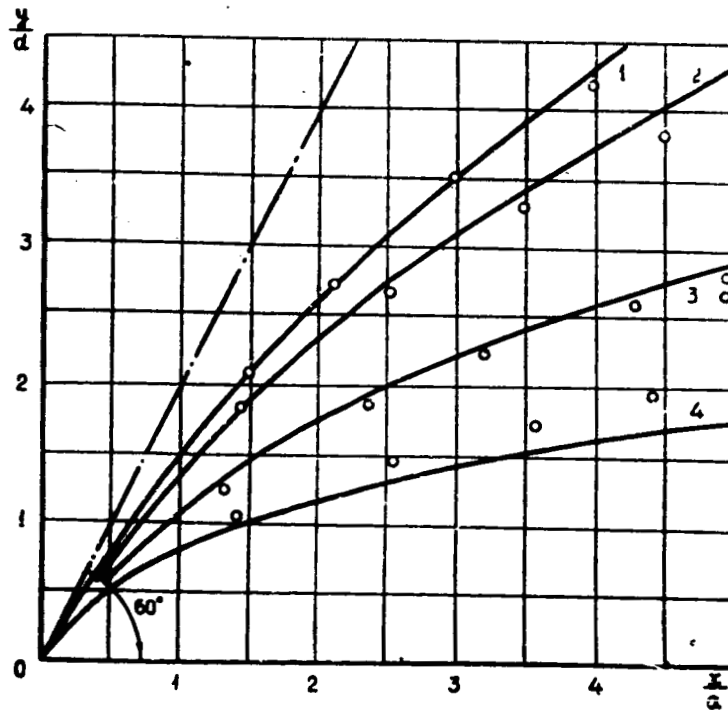


Figure 4. Axes of the jets which enter the flow at an angle $\alpha_0 = 60^\circ$. The solid lines show the computed axes:
 1 - $q_2/q_1 = 24.8$,
 2 - $q_2/q_1 = 16$,
 3 - $q_2/q_1 = 6.4$,
 4 - $q_2/q_1 = 2.5$.
 The circles designate the coordinates which correspond to axes determined experimentally (according to data contained in reference 1.)

Figure 4 shows the axes of jets computed by means of equation (7) when $\alpha_0 = 60^\circ$, $C_n = 3.0$ for various values of q_2/q_1 . The same graph contains the experimental values of point coordinates corresponding to maximum velocity in the transverse cross sections of the jet (ref. 1). We can see that when $C_n = 3.0$ the computed axes of the drifting jets are in good agreement with the experimental ones. This confirms that the adopted simplifying assumptions are valid.

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где

$$C_n = 2 \frac{C_x}{\pi} \frac{q_2}{q} \frac{d_0}{d}, \quad (3)$$

q_1, q_2, q — скоростные напоры сносящего потока струи в начальном и текущем сечениях, соответственно.

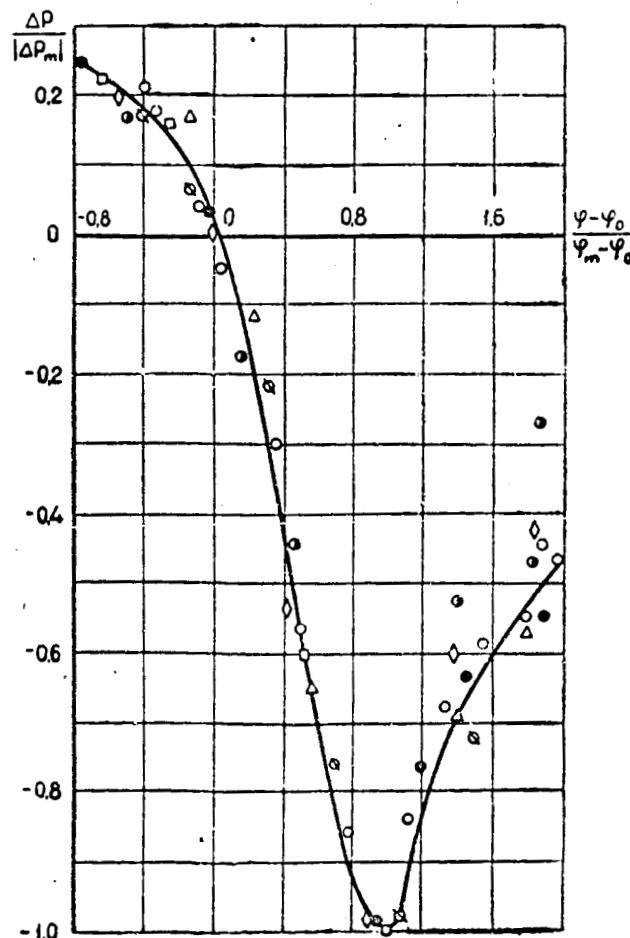
Заменим

$$\sin \alpha = \frac{y'}{(1 + y'^2)^{0.5}}, \quad R = \frac{(1 + y'^2)^{1.5}}{y''}.$$

Тогда уравнение (2) будет иметь вид:

$$\frac{C_n}{d_0} \frac{q_1}{q_2} y'^2 = \frac{y''}{(1 + y'^2)^{0.5}}. \quad (4)$$

Перед интегрированием (4) рассмотрим внимательнее величину C_n , входящую в это уравнение. В выражение для C_n (3), помимо начальных параметров q_2, d_0 , входят три, вообще говоря, переменные величины: C_x, q и d . Известно, что по мере удаления от устья уменьшение скоростного напора струи сопровождается увеличением



Фиг. 2. Распределение давления вокруг начального сечения струи в безразмерных координатах:

$q_2/q_1 = 15 \quad 10 \quad 5 \quad 3.75 \quad 2.5 \quad 2.0 \quad 1.5$

ее сечения, так что влияние параметров q и d на величину C_n в какой-то мере компенсируется. Иначе говоря, можно положить:

$$C_n \approx 2 \frac{C_x}{\pi}. \quad (5)$$

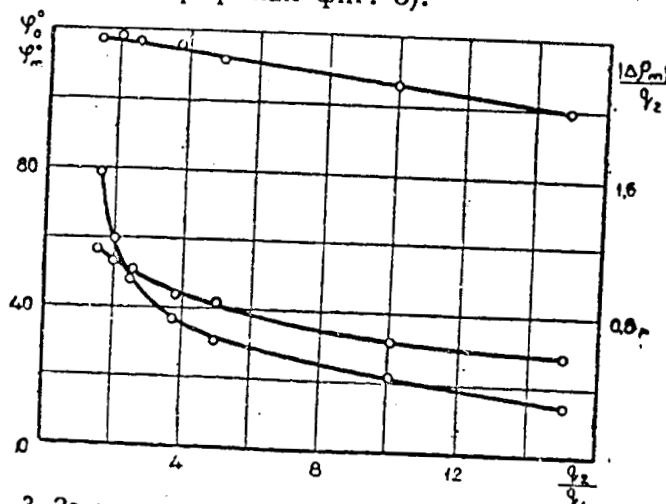
Распределение давления вокруг струи в сносящем потоке, определяющее величину C_x , как было показано в работе [1] для устья струи, входящей в поток под углом $\alpha_0 = 90^\circ$, является функцией отношения скоростных напоров струи и потока. На фиг. 2 приведен профиль распределения давления вокруг начального сечения струи в безразмерных координатах:

$$\frac{\Delta p}{|\Delta p_m|} = \frac{\varphi - \varphi_0}{\varphi_m - \varphi_0}.$$

Здесь Δp_m — максимальное разрежение вокруг струи,
 Δp — статическое давление, избыточное относительно давления в невозмущенном сносящем потоке,
 φ — угловая координата текущей точки окружности вокруг струи,
 φ_0 — угловая координата точки, давление в которой равно давлению в сносящем потоке,
 φ_m — угловая координата точки, давление в которой равно Δp_m .

В исследованном в работе [1] диапазоне отношений $1,5 \leq q_2/q_1 \leq 15$ экспериментальные точки практически легли на одну кривую. В заданной части струи при $\varphi > \varphi_m$ разброс точек больше, чем при $\varphi < \varphi_m$, а при $q_2/q_1 \leq 2$ наблюдается систематический отход от кривой — следствие усиливающегося влияния стенки. При большой скорости в сносящем потоке газ, выходящий струей из сопла, „прилипает“ к стенке. Универсальность в распределении давления должна очевидно нарушаться и при очень малых скоростях сносящего потока, так как в предельном случае, соответствующем свободной струе, избыточные давления во всех точках сечения одинаковы и равны нулю.

При пользовании универсальным профилем распределения давления вокруг струи необходимо знать зависимости $|\Delta p_m|/q_2$, φ_0 и φ_m от отношения q_2/q_1 . (Эти зависимости, определенные экспериментально, приведены на графиках фиг. 3).



Фиг. 3. Зависимость максимального безразмерного разрежения в начальном сечении струи $|\Delta p_m|/q_2$ и координат φ_0 и φ_m от отношения скоростных напоров в струе и в потоке

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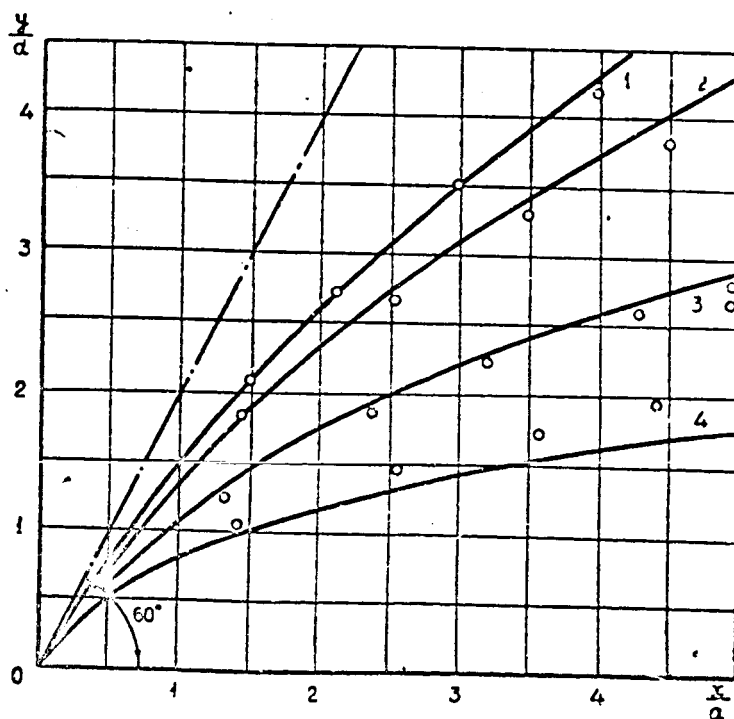
Как видно из графиков, с изменением отношения q_2/q_1 происходит значительная деформация эпюры давления вокруг струи. Обработка приведенных опытных данных показывает, однако, что при изменении q_2/q_1 от 1,5 до 15 величина C_x изменяется сравнительно мало: ¹⁾ от 3,5 до 4,5. Следует также отметить, что в [1] распределение давления вокруг устья струи определялось путем дренажа плоскости среза сопла на расстоянии 1,5 мм от окружности выходного отверстия. Поэтому можно ожидать несколько большего фактического значения C_x , а именно — порядка 4,2 + 5,2. При этом вероятные значения C_n , согласно (5), равны 2,7 + 3,3.

Приняв в первом приближении величину C_n постоянной по длине струи и одинаковой для разных отношений q_2/q_1 , можно без труда проинтегрировать дифференциальное уравнение (4). В результате получим:

$$\frac{y}{d_0} = \frac{1}{C_n} \cdot \frac{q_2}{q_1} \cdot \ln 2 \left[\sqrt{\left(C_1 + C_n \frac{q_1}{q_2} \frac{x}{d_0} \right)^2 - 1} + C_1 + C_n \frac{q_1}{q_2} \frac{x}{d_0} \right] + C_2. \quad (6)$$

Постоянные интегрирования C_1 , C_2 найдем из граничных условий:

$$\text{при } x=0, \quad y=0, \quad y' = \operatorname{tg} \alpha_0.$$



Фиг. 4. Оси струй, входящих в поток под углом $\alpha_0 = 60^\circ$. Сплошными линиями показаны расчетные оси:

- 1 — $q_2/q_1 = 24,8$, 2 — $q_2/q_1 = 16$,
3 — $q_2/q_1 = 6,4$, 4 — $q_2/q_1 = 2,5$.

Кружками обозначены координаты соответствующих осей, определенные экспериментально (по данным работы [1]).

¹⁾ Большая величина C_x объясняется наличием разрежения за сносимой струей, значительно превышающего то, которое создается на поверхности твердого цилиндра при обтекании его потоком.

Отсюда

$$C_1 = \frac{1}{\sin \alpha_0}, \quad C_2 = -\frac{1}{C_n} \cdot \frac{q_2}{q_1} \ln 2 \operatorname{ctg} \frac{\alpha_0}{2}.$$

Подставив значения C_1, C_2 в уравнение (6), окончательно получим:

$$\frac{y}{d_0} = \frac{1}{C_n} \frac{q_2}{q_1} \ln \operatorname{tg} \frac{\alpha_0}{2} \left[\sqrt{\left(\frac{1}{\sin \alpha_0} + C_n \frac{q_1}{q_2} \frac{x}{d_0} \right)^2 - 1} + \frac{1}{\sin \alpha_0} + C_n \frac{q_1}{q_2} \frac{x}{d_0} \right]. \quad (7)$$

На фиг. 4 изображены оси струй, рассчитанные по формуле (7) при $\alpha_0 = 60^\circ$, $C_n = 3,0$ и разных значениях q_2/q_1 . На этот же график нанесены экспериментальные значения координат точек, соответствующих максимальной скорости в поперечных сечениях струй [1]. Видно, что при значении $C_n = 3,0$ расчетные оси сносимых струй удовлетворительно согласуются с экспериментальными. Последнее свидетельствует о приемлемости принятых упрощающих предположений.

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V. Z.

The article

abstract

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Recently, efforts have been made to compute theoretically the form of the axis corresponding to plane fan and twin jets in free, drifting flows based on the consideration of forces which act on the elementary region of the jet (refs. 2 and 3).

We shall show that this approach to the solution of the problem on the inflection of the jet leads to satisfactory results in the case when we have a single drifting jet whose initial cross section is round and which propagates in a homogeneous infinite subsonic flow.

For this purpose we isolate a jet element having a length dl at some distance from the mouth of the jet, which is situated in the xoz plane (fig. 1). We shall assume that the aerodynamic force which bends the jet is proportional to the velocity head of the normal velocity component of the drift-producing flow and is counterbalanced by the centrifugal force. Then the condition for the radial equilibrium of the jet element dl may be written in the form

$$dN = dQ, \quad (1)$$

where $dN = C_x \frac{\rho_1 \omega_1^2}{2} \sin^2 \alpha dl$, $dQ = \frac{\rho \omega^2}{R} \frac{\pi d^2}{4} dl$ are the aerodynamic and centrifugal forces. Substituting the expressions for the forces acting on the element dl into equality (1) we obtain

$$C_n \frac{q_1}{q_2} \sin^2 \alpha = \frac{d_0}{R}, \quad (2)$$

where

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*Numbers given in margin indicate pagination in original foreign text.

$$C_n = 2 \frac{C_x}{\pi} \frac{q_2}{q} \frac{d_0}{d}, \quad (3)$$

q_1 , q_2 , q are the velocity heads of the flow producing the drift of the jet at the initial and running cross sections respectively. We carry out the substitutions

$$\sin \alpha = \frac{y'}{(1+y'^2)^{0.5}}, \quad R = \frac{(1+y'^2)^{1.5}}{y''}.$$

Then equation (2) will have the form:

$$\frac{C_x}{d_0} \frac{q_1}{q_2} y'^2 = \frac{y''}{(1+y'^2)^{0.5}}. \quad (4)$$

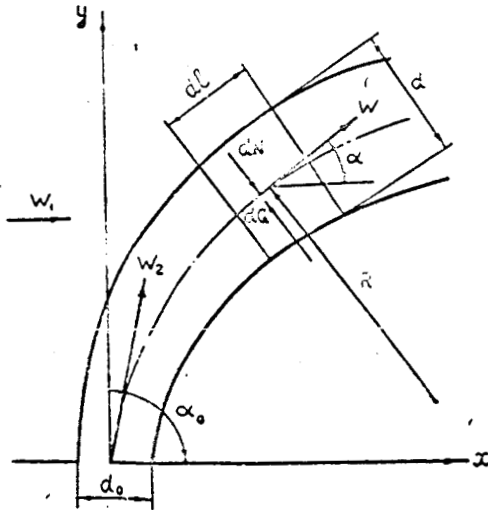


Figure 1. Schematic of the jet in the drifting flow.

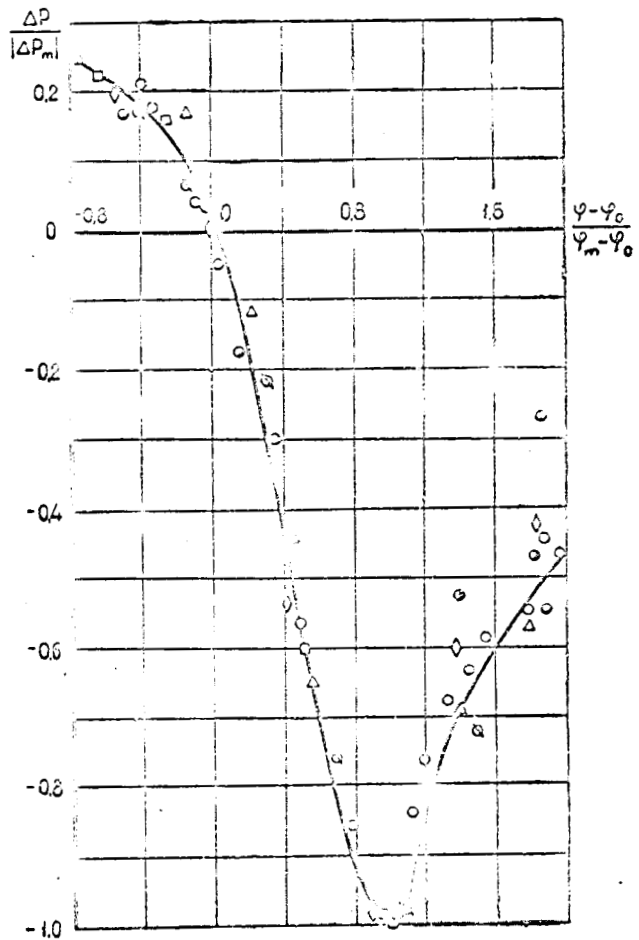


Figure 2. Pressure distribution around the initial cross section of the jet in terms of dimensionless coordinates:

$$q_2/q_1 = 1.5 \quad 2.0 \quad 2.5 \quad 3.75 \quad 5 \quad 10$$

Before integrating (4) we consider carefully the quantity C_n contained in this equation. In the expression for C_n (3), three generally speaking, three variable quantities C_x , q and d are contained in addition to parameters q_2 , d_0 .

It is known that as we move away from the mouth the decrease in the velocity head of the jet is accompanied by an increase in its cross section so that the effect of parameters q and d on the quantity C_n is compensated for to some extent. In other words, we may assume that: /102

$$C_n \approx 2 \frac{C_x}{\pi}. \quad (5)$$

As shown in reference 1, the pressure distribution around the jet in the drifting flow, which determines the quantity C_x when the mouth of the jet enters the flow at an angle $\alpha_0 = 90^\circ$, is a function of the ratio of jet and flow impact pressures. Figure 2 shows the pressure distribution profile around the initial section of the jet in the dimensionless coordinates:

$$\frac{\Delta p}{|\Delta p_m|}, \quad \frac{\varphi - \varphi_0}{\varphi_m - \varphi_0}.$$

Here Δp_m is the maximum rarefaction around the jet,

- Δp is the static pressure in excess of the pressure in the unperturbed drifting flow,
- φ is the angular coordinate of the considered point on the surface around the jet,
- φ_0 is the angular coordinate of the point at which the pressure is equal to the pressure of the drifting flow,
- φ_m is the angular coordinate of the point where the pressure is equal to Δp_m .

For the range of ratios $1.5 \leq q_2/q_1 \leq 15$ investigated in reference 1, the experimental points lie practically on one curve. In the rear part of the jet when $\varphi > \varphi_m$ the spread of the points is larger than when $\varphi < \varphi_m$, and in case $q_2/q_1 \leq 2$ there is a systematic departure from the curve due to the increased influence of the wall. For large velocity in the drift flow, the jet which exists from the nozzle "adheres" to the wall. The universal nature of pressure distribution must obviously be violated also when the velocities of the drift flow are very small, because as in the limiting case corresponding to the free jet, the excess pressures at all points of the cross section are the same and are equal to zero.

When using the universal profile for the pressure distribution around the jet, we must know the relationships $|\Delta p_m|/q_2$, φ_0 and φ_m as a function of the ratio q_2/q_1 . (These relationships, which have been determined experimentally, are shown in the graphs of figure 3.)

As we can see from the graphs, when the ratio q_2/q_1 changes, there is a /103 substantial deformation of the pressure diagram around the jet. The processing of presented experimental data shows, however, that when q_2/q_1 varies from 1.5 to 15, the value of C_x changes relatively little:¹ from 3.5 to 4.5. It should also be pointed out that in reference 1 the pressure distribution around the mouth of the jet was determined by draining the nozzle cutoff plane at a distance of 1.5 mm from the circumference of the output hole. Therefore we can expect a somewhat higher actual value for C_x , specifically a value of the order of 4.2 - 5.2. In this case the probable values of C_n according to (5) are equal to 2.7 - 3.3.

Assuming, in the first approximation, that the magnitude of C_n is constant along the length of the jet and is the same for different ratios q_2/q_1 , we can easily integrate the differential equation (4). As a result we obtain:

$$\frac{y}{d_0} = \frac{1}{C_n} \cdot \frac{q_2}{q_1} \cdot \ln 2 \left[\sqrt{\left(C_1 + C_n \frac{q_1}{q_2} \frac{x}{d_0} \right)^2 - 1} + C_1 + C_n \frac{q_1}{q_2} \frac{x}{d_0} \right] + C_2. \quad (6)$$

The constants of integration C_1 , C_2 are obtained from the boundary conditions:

$$\text{for } x = 0, y = 0, y' = \operatorname{tg} \alpha_0.$$

It follows that

$$C_1 = \frac{1}{\sin \alpha_0}, \quad C_2 = -\frac{1}{C_n} \cdot \frac{q_2}{q_1} \ln 2 \operatorname{ctg} \frac{\alpha_0}{2}.$$

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Substituting the values of C_1 , C_2 into equation (6), we finally obtain:

$$\frac{y}{d_0} = \frac{1}{C_n} \cdot \frac{q_2}{q_1} \ln \operatorname{ctg} \frac{\alpha_0}{2} \left[\sqrt{\left(\frac{1}{\sin \alpha_0} + C_n \frac{q_1}{q_2} \frac{x}{d_0} \right)^2 - 1} + \frac{1}{\sin \alpha_0} + C_n \frac{q_1}{q_2} \frac{x}{d_0} \right]. \quad (7)$$

¹The large value of C_n is explained by the presence of rarefaction behind the drift jet which substantially exceeds the one produced on the surface of a solid cylinder when there is flow around it.

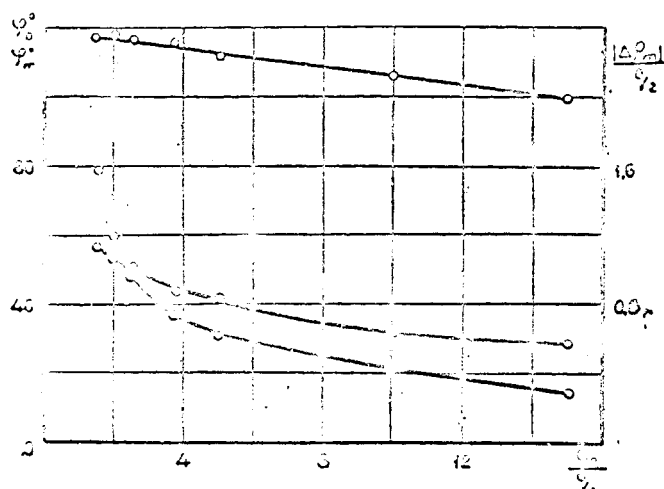


Figure 3. Variation in the maximum dimensionless rarefaction at the initial cross section of the jet $|\Delta p_m|/q_2$ and in the coordinates φ_0 and φ_m as a function of the ratio of impact heads in the jet and in the stream.

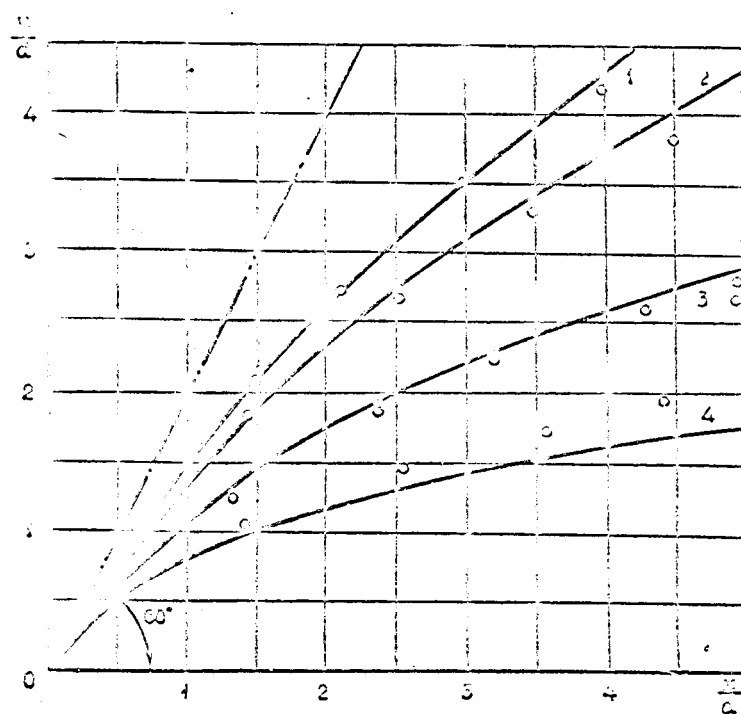


Figure 4. Axes of the jets which enter the flow at an angle $\alpha_0 = 60^\circ$. The solid lines show the computed axes:
 1 - $q_2/q_1 = 24.8$,
 2 - $q_2/q_1 = 16$,
 3 - $q_2/q_1 = 6.4$,
 4 - $q_2/q_1 = 2.5$.
 The circles designate the coordinates which correspond to axes determined experimentally (according to data contained in reference 1.)

Figure 4 shows the axes of jets computed by means of equation (7) when $\alpha_0 = 60^\circ$, $C_n = 3.0$ for various values of q_2/q_1 . The same graph contains the experimental values of point coordinates corresponding to maximum velocity in the transverse cross sections of the jet (ref. 1). We can see that when $C_n = 3.0$ the computed axes of the drifting jets are in good agreement with the experimental ones. This confirms that the adopted simplifying assumptions are valid.

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